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Technical Report No. 70

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THEORETICAL BRILLOUIN (k−β) DIAGRAM FOR MONOPOLE AND DIPOLE ARRAYS AND THEIR APPLICATION TO LOG-PERIODIC ANTENNAS

by

R. Mittra and K. E. Jones

Contract No. AF33(657)-10474
Hitch Element Number 62405484
760D — Project 6278, Task 6278-01

April 1963

Sponsored by
AERONAUTICAL SYSTEMS DIVISION
WRIGHT-PATTERSON AIR FORCE BASE, OHIO
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(7) NA
(8) NA
(9) NA

Antenna Laboratory

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(11) apr 63,
(12) 18p.
(13) NA
(14) TR70
(18-19) NA
(20) U
(21) NA

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(15) Contract No. AF33(657)-10474
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Electrical Engineering Research Laboratory
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University of Illinois
Urbana, Illinois

Theoretical Brillouin (k-β) Diagram for Monopole and Dipole Arrays and their Application to Log-Periodic Antennas

R. Mittra[+] and K. E. Jones[++]

SUMMARY

In this paper we derive the characteristic equation for the complex propagation constant in a transmission line periodically loaded with dipole elements. is derived This equation is solved for real and imaginary parts of the mode propagation constants. The effect of mutual impedance between antenna elements is included in the formulation and calculation. Theoretical calculations are compared with experimental results reported by Mayes and Ingerson. The ~~usefulness~~ and limitation of the Brillouin ~~(k-β)~~ diagram for analyzing LP structures is discussed briefly.

INTRODUCTION

During the recent years a new concept of broadband antenna design has been introduced under the name of Log-periodics or LP and the usefulness of the concept has been demonstrated by several practical designs[1,2] of the same type. From the definition of LP structures it is obvious that the electromagnetic properties of an infinite LP structure repeat at $f_o\tau^n$, where f_o is a reference frequency and n is a positive or negative integer. However, this definition says nothing about the behavior of the structure at the in-between frequencies. In order to be successful, a practical antenna which necessarily has finite dimensions, must not only have LP properties in the design range but should also exhibit under continuous sampling, little variation in pattern and impedance throughout this frequency range. How exactly one is to assure this behavior when designing a LP antenna is a question which has not been answered satisfactorily to date. The LP design has largely been an art, an application of empiricism based on intuition, and not really a science.

Most of the papers written on the subject are experimental in nature and the theoretical development of the LP antenna design is very much in its infancy.

A significant theoretical work in this line is by Carrel[3] who has analyzed the LP dipole antenna. His analysis of this particular problem is very thorough but his results are clearly not meant to be applicable directly to other LP structures. Some LP antennas may be represented in terms of radiating elements loading a transmission line in a LP manner and a study of such non-uniformly loaded lines has recently been reported by Mittra[4] and Jones and Mittra[5]. Again this line of attack, although quite useful for some LP antennas, does not cover all possible configurations. An approach which views the LP device as a tapered version of a uniform periodic structure was first suggested by Mayes, Deschamps and Patton[6]. This work considered periodic structures or gratings which have the following propagation characteristics as a function of frequency. The grating is assumed to support bounded waves or surface waves up to a certain frequency, implying that the propagation constant β along the structure is greater than k, the free space wave number, up to this frequency. As the frequency is increased further the wave becomes fast, i.e., β becomes less than k. Now the value of β relative to k gives the direction of the main lobe of the pattern through the simple relation.

$$\cos\theta = \beta/k \qquad \text{for} \quad \beta < k$$

This picture although simple and quite useful, does not cover the case of all periodic structures but only those which support a wave characterized by a slowness factor which remains relatively unchanged as a function of frequency. Only the periodic structures belonging to the helix family, which includes the zigzag, are known to possess such characteristics.

A portion of a recent report by Mittra[7] includes a brief sketch on the use of the k-β or Brillouin diagram of periodic structures to predict the performance of its LP counterpart. A more elaborate discussion which generalizes their first work on the same topic has recently been published by Mayes[8] et al. Oliner[9] has also considered this aspect of the LP study in a recent paper. It seems that viewing the LP as a tapered periodic structure is the only general approach available so far for theoretically studying the log-periodics. It must be emphasized, however, that the arguments behind this approach so far have largely been heuristic. A solid theoretical foundation and a more exact formulation for the LP

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structure has yet to be worked out. The authors are currently looking into this problem in some detail. It is also quite important to study the limitations of this approach in order to estimate the range of its usefulness.

At present there are extremely few periodic structures for which the complete k-β diagram, exhibiting both the real and complex roots has been calculated. Moreover, none of the above has been related to the corresponding LP structures except on a qualitative basis. We propose to calculate the complete k-β diagram of the dipole loaded transmission and relate the calculations to the results obtained by Carrel for the LP antenna. In addition, we compare our results with experimental measurements on the uniform periodic dipole and monopole structures. These have been reported by Mayes and Ingerson[10]. Before discussing the dipole problem, however, we shall present a brief discussion on the k-β diagram and its significance to the LP analysis.

THE BRILLOUIN DIAGRAM AND ITS SIGNIFICANCE TO LP ANALYSIS

The Brillouin diagram of a periodic structure relates the propagation constant β along the structure to the free space wave number k and hence is often referred to as the k-β diagram. The characteristics of the k-β diagram for a periodic structure vary widely depending upon the nature of the structure. As an illustration of some of the different types of diagrams, reference is made to Figure 1. One notes that the three examples shown have quite different characteristics, and although it may be possible to group different periodic structures into several different broad categories it is not possible to draw hard and fast lines between them. The three examples shown may however be considered as belonging to three typically different groups. The first one, the helix, may be thought of as a constant slowness type of structure mentioned before. Figure 1a gives a sketch of the solutions for two modes of propagation which may exist simultaneously on the helix. Out of these, the one on the left hand side turns complex above a certain frequency called the turnover frequency. The phase of the wave in the neighborhood of the turnover region is suitable for backfire radiation. When this structure is tapered, it will support bound waves for smaller diameters of the helix and will eventually reach a region where the wave on the structure will attenuate because of radiation. This is assuming that the amplitude of excitation of the second mode represented by the right line is relatively small in the region of interest.

In the corrugated surface antenna, no complex solutions of the type possessed by the helical structure have been obtained. The only complex solutions found have a real part of equal to π and theoretically, for an infinite structure, this solution represents a filter type cutoff mode with no radiating properties. This is not very desirable and the tapered version of the corrugated surface is not very suitable for LP design without some basic modifications. Further details on this structure may be found in an accompanying paper, "The Letter Rack Antenna" by Mittra and Wahl[11].

The third type of diagram for the stub-loaded monopole array (see Figure 1c) has the exceptional feature that its k-β characteristic seems to be discontinous as a function of frequency. Although the experimental diagram in Figure 1c does not show this, it is conjectured that there exists more than one mode on the structure although only one of these is perhaps predominant at any one frequency. The switching of the curve from the right hand side to the negative β region may well be due to switching of modes. At any rate this type of structure may also be tapered to give a wideband performance although the design may be quite critical.

It should be realized that the solution of the source problem, which gives the relative amplitude coefficients of the various modes, is not easy even for a periodic structure. Even if this problem is solved for the periodic case, the solution cannot be easily related to the corresponding one for a LP structure. This therefore is one of the fundamental difficulties in applying the k-β diagram of a periodic structure directly to the case of the corresponding LP one. Some investigations are now underway toward resolving this problem.

The multimode problem is quite prominent in the uniform monopole and dipole arrays as we shall see later.

Following this general introduction we shall now discuss the problem of the monopole and dipole loaded transmission lines in detail.

DERIVATION OF THE CHARACTERISTIC EQUATION

The first step toward calculating the k-β diagram is the formulation of the characteristic equation for β. This is to be outlined in the following.

Consider a transmission line loaded periodically with a network describable by an admittance matrix [Y] also having periodic properties, as shown in Figure 2. It is assumed that y_{mn}, the mutual admittance between the nodes m and n is independent of m and n with $|m-n|$ held constant.

It is obvious from the above that the entire structure is periodic and has a period d.

Writing the node equation say for node 1, one has using the symmetry property of the admittances, viz. $y_{1q} = y_{1-q}$

$$0 = \;--+y_{13}V_{-2}+(y_{12}+j\ \mathrm{cosec}\ kd)V_{-1}$$

$$+(-2\ j\ \cot\ kd + y_{11})V_o+(y_{12}+j\ \mathrm{cosec}\ kd)V_1$$

$$+ y_{13}V_2 + --- \quad (1)$$

where V_m's are the node voltages and y_{11} is the self admittance of the loading network for node 1. From the periodic nature of the structure we have,

if $$\frac{V_1}{V_o} = e^{-j\beta' d}$$

$$\beta' = \beta + j\ \alpha$$

then $$\frac{V_p}{V_q} = e^{-j(p-q)\ \beta' d} \quad (2)$$

where p and q are arbitrary.
Using the above relation in Equation (1) we derive after some simplification

$$0 = (-2j\ \cot\ kd + y_{11})+2(y_{12}+j\ \mathrm{cosec}\ kd)\cos\ \beta' d$$

$$+ 2y_{13}\ \cos\ 2\ \beta' d + \;-- \quad (3)$$

or

$$\cos\ \beta' d = \cos\ kd + \frac{j\ y_{eq}}{2}\sin\ kd$$

$$(4)$$

$$y_{eq} = y_{11} + 2y_{12}\ \cos\ \beta' d + 2y_{13}\ \cos\ 2\beta' d + ---$$

Equation (4) is the desired characteristic equation for the configuration shown in Figure 2. Note that if $y_{1m} = 0$, for $m \geq 1$, then Equation (4) reduces to the well-known periodically loaded transmission line characteristic eauation.

Now consider a slightly different formulation in terms of the impedance matrix parameters of the loading network. Let the impedance matrix [Z] of the periodic loading structure be represented by

$$[Z] = \begin{bmatrix} -- & z_{12} & z_{11} & z_{12} & z_{13} & -- & & \\ & -- & z_{13} & z_{12} & z_{11} & z_{12} & z_{13} & -- & \\ & & & -- & z_{13} & z_{12} & z_{11} & z_{12} & z_{13} & -- \end{bmatrix} \quad (5)$$

Note that any row of the [Z] matrix may be related to any other row by a simple shift. This of course follows from the periodicity property required here of the loading device. In order to use Equation (4) one would like to calculate y_{eq} in terms of the parameters of the [Z] matrix. To obtain the above, use the n^{th} equation from the matrix relation

$$\{V\} = [Z]\ \{I\}$$

and obtain

$$V_n = \;--+z_{13}\ I_{n-2} + z_{12}\ I_{n-1} + z_{11}\ I_n$$

$$+ z_{12}\ I_{n+1} + z_{13}\ I_{n+2} + -- \quad (6)$$

Use the periodicity property of the structure in connection with Equation (6) and obtain

$$\frac{V_n}{I_n} = z_{11} + 2z_{12}\ \cos\ \beta' d + 2z_{13}\ \cos\ 2\beta' d + -- \quad (7)$$

where

$$\frac{V_m}{V_n} = e^{-j(m-n)\ \beta' d}$$

If we start with the admittance Matrix [Y] we may similarly derive an alternate expression for V_n/I_n, viz.,

$$\frac{I_n}{V_n} = y_{11} + 2y_{12}\cos\ \beta' d + 2y_{13}\cos\ 2\beta' d + --- \quad (8)$$

It immediately follows from Equations (7) and (8) that y_{eq} defined in connection with Equation (4) may be expressed as

$$y_{eq} = \frac{1}{z_{11} + 2z_{12}\cos\ \beta' d + 2z_{13}\cos 2\beta' d + --} \quad (9)$$

This is an important relation which permits one to evaluate the desired quantity y_{eq} in terms of the impedance parameters of the loading structure. It follows that the characteristic equation in terms of the impedance parameters of the loading network is

$$\cos\ \beta' d = \cos\ kd - \left(\frac{j}{2}\right)\frac{\sin\ kd}{z_{11} + 2z_{12}\ \cos\ \beta' d + 2z_{13}\ \cos\ 2\beta' d + --} \quad (10)$$

Equation (10) gives a suitable form of the characteristic equation for a dipole loaded transmission line. This is because it is considerably simpler to calculate the z-parameters of an array of dipoles as compared to its admittance parameters. The Equation (10) has also been independently derived by Laxpati*.

An alternative way of handling the problem is as follows. One may obtain the parameters of the [Y] matrix by directly inverting the [Z] matrix. We shall illustrate the procedure by first considering the simple case in which we assume that $z_{1n} = 0$ for $n > 2$. Then [Z] becomes

$$[Z] = \begin{bmatrix} -- & & & & & & \\ 0 & z_{12} & z_{11} & z_{12} & 0 & -- & \\ 0 & 0 & z_{12} & z_{11} & z_{12} & 0 & 0 \\ & 0 & 0 & z_{12} & z_{11} & z_{12} & 0 \\ & & & & & -- & \end{bmatrix} \tag{11}$$

Now let

$$[Y] = \begin{bmatrix} y_{13} & y_{12} & y_{11} & y_{12} & -- & & & \\ -- & y_{13} & y_{12} & y_{11} & y_{12} & y_{13} & & \\ -- & & y_{13} & y_{12} & y_{11} & y_{12} & y_{13} & -- \end{bmatrix} \tag{12}$$

Using [Z] [Y] = [1] = unit matrix, one may derive the following approximate equations for the parameter y_{11} and y_{12} in terms of the z_{11} and z_{12}:

$$\begin{aligned} z_{11}\, y_{11} + 2z_{12}\, y_{12} &= 1 \\ z_{11}\, y_{12} + z_{12}\, y_{11} &= 0 \end{aligned} \tag{13}$$

Note that we have neglected the higher order y_{1n}'s and also the higher order equations required to obtain the inverse of [Z]. With this approximation it is a simple matter to evaluate y_{11} and y_{12} in terms of z_{11} and z_{12} from Equation (13). The above procedure may be extended to include the higher order mutual terms. That this method is only an approximate one is fairly obvious. It may be verified, however, that the use of the admittance parameters reduces the order of the characteristic equation by one from that of the impedance parameter equation of the type (10) when the same number of mutual terms are included and there may be some merit to using this procedure.

* Unpublished work

It is not difficult to show that this procedure would yield exactly the same expression for y_{eq} as given in Equation (9) if the inversion process is carried out exactly, as it of course must.

For the dipole array the z-parameters may be calculated by using the formula given below. The expression may be derived by using the induced-emf method. The mutual impedance between two antennas is expressed by

$$z_{12} = \frac{60}{1-\cos 2kd}\Big\{e^{j2kh'}[K(u_o)-2K(u_1)] + e^{-2jkh'}[K(V_o)-2K(V_1)] + 2[K(u_o')-K(u_1)-K(V_1)] + 2K(u_o')\,[1+\cos 2kh']\Big\}^* \tag{14}$$

The star indicates the complex conjugate of the expression in the braces. Here

$$K(x) = Ci\,(x) + j\,Si(x)$$

where K(x) and Ci(x) are the cosine and sine integral functions of the argument x. also

$$u_o = k\left[\sqrt{d^2 + 4h'^2} - 2h'\right]$$

$$V_o = k\left[\sqrt{d^2 + 4h'^2} + 2h'\right]$$

$$u_o' = kd$$

$$u_1 = k\left[\sqrt{d^2 + h'^2} - h'\right]$$

$$V_1 = k\sqrt{d^2 + h'^2} + h'\,\Big]$$

h' = half length of the dipole elements. d = spacing between the elements. Equation (14) is of the same type as used by Carrel except that because of unequal lengths of the elements in the LP antenna he had to use a more general formula applicable to such cases.

The self impedance z_{11} is calculated by simply setting $d = \sqrt{2a}$ in Equation (14), where a is the radius of the antenna element.

When the alternate dipoles in an array are transposed, $z_{1n}^{(t)}$ the mutual impedances between the elements 1 and n in the resulting structure is simply given by

$$z_{1n}^{(t)} = (-)^{n+1} z_{1n} \tag{15}$$

The characteristic equation for the alternately transposed structure is simply obtained by modifying the z-parameters appearing in the equation for β' according to Equation (15).

SOLUTION OF CHARACTERISTIC EQUATION

In this section we discuss the solution of the characteristic equation for β' for a loaded transmission line. Let us start with Equation (10) and assume that all z_{1n}'s are zero for $n > q$. After going through some algebra it is possible to derive an equation in terms of cos $n\beta' d$ which reads

$$\sum_{n=o}^{q+1} C_n \cos n\beta' d = o \tag{16}$$

where C_n's may be expressed in terms of the z-parameters. Using the transformation $t = e^{j\beta' d}$, it is possible to derive an algebraic equation in t^2 which reads

$$\sum_{n=o}^{q} d_n t^{2n} = o \quad , \qquad t = e^{j\beta' d} \tag{17}$$

The coefficients of the above equation are complex numbers in general. The roots of this polynomial equation may be sought by using numerical techniques. The advantage of Equation (17) over Equation (16) is that whereas Equation (16) is transcendental, Equation (17) is algebraic and can be handled on the digital computer using standard library routines.

One question immediately arises in connection with Equation (15). In general, the above equation has (q+1) solutions where q depends on the number of mutual impedances taken to be non-zero. Does this imply that as the number q is increased indefinitely the number of solutions become infinitely many? The physical implication of this will be that an infinite number of characteristic modes may be supported by the structure. It is worthwhile examining this statement to see if it is true or not. This question may be answered by going back to Equation (14) which is in fact the source of Equation (15). The left hand side of Equation (14), say $f(\beta')$, is a periodic function of real β'. If β' is real, one would expect only a finite number of zero crossings of $f(\beta')$ in a period of β', i.e., for $o \leq \beta' \leq 2\pi/d$. It is not possible to make any such definite statements when β' is complex, however. In any case, whether or not the modes with the complex β's may be supported by the structure may be answered only by solving a source problem. The complex solutions of the eigen-value equation of an infinite periodic structure characterize approximate and **alternative** representation of continous spectrum of spatial harmonics and must be interpreted in this light.

One usually finds that the imaginary parts of many of these solutions which determines the attenuation per cell is so large that they play little part in contributing to the current distribution produced by a given source. In addition to this there are a whole class of complex roots which are physically inadmissible because they do not satisfy the consistency conditions for the attenuation and phase constants. For details on this condition the reader is referred to Oliner[9]. Once again it must be remembered that the complex solutions merely serve to represent the solution to a source problem in approximate manner.

NUMERICAL CALCULATIONS AND COMPARISON WITH EXPERIMENTS

Single Mutual Term

Although computations have been carried out for the k-β diagrams for several different parameters of the dipole array, we shall only report the calculations for a choice of dimensions which corresponds closely to one of the LP arrays investigated by Carrel.

Let us start with the discussion of the case of one mutual impedance and assume that the effect of the higher order mutual terms is negligible. Assume further that the approximate Equation (13) is adequate for calculating y_{11} and y_{12}. For this approximate case, Equation (4) is quite convenient to work with, and we neglect all the y_{1n}'s for $n > 2$ in the expression for y_{eq}. The propagation constant β may be calculated with little difficulty and we get only a single solution for β under the present approximation. Computations were made for the following parameters

$$d = .1, \; h = 1.0, \; h/a = 177$$

a = radius of dipole element

for both the unreversed and the reversed cases, the schematics for which are shown in Figure 3a and 3c respectively. Note that the characteristics of the monopole array over the ground plane (see Figure 3b) may be deduced from that of the unreversed dipole array from symmetry considerations.

The propagation constant β $(= \beta + i\alpha)$ was calculated as a function of the free space wave number k. A plot of kd vs β d and αd is shown in Figure 4a and 4b for the unreversed and reversed case, respectively. Recall that for the reversed case one changes the sign of the mutual impedance z_{12} in the expression for the characteristic equation.

Figures 4a and 4 b also show the experimental curves by Mayes and Ingerson[10]. Note the close correspondence of the theoretical and experimental curves for both the structures. The major deviation between the experiment and the theory seems to occur in the neighborhood of the dipole resonance where the experimental curve penetrates much farther towards a larger β. An explanation for this discrepancy will be offered later when higher order mutual terms will be considered.

The results for the uniform case may be applied to the LP structure by using a perturbational analysis. One calculates the βd for the local kd, where d now changes from cell to cell and uses the complex value of βd to calculate the attenuation and phase shift due to that cell. The results may then be compared with the corresponding values for the LP case. The above computations were carried out for the case of alternately reversed dipoles and they are plotted in Figure 5 which shows the feeder voltage amplitude and phase plotted as a function of distance along the lines. The theoretical curves in Figure 4 were calculated using d = 0.112h to correspond to Carrel's choice of parameters. The calculated points are plotted on Carrel's experimental curves and close correspondence between the two is indeed striking, considering the approximations which were used to calculate the theoretical points.

<u>Three Mutual Terms</u>

Computations were also carried out to study the modification of the results when the higher order mutual terms are included. The dipole spacing d was now chosen to be 0.112h in order to correspond exactly to Carrel's case and other parameters were kept unchanged.

We now use Equation (10) and include three mutual impedances, z_{12}, z_{13} and z_{14} which may be calculated by using Equation (14). The characteristic equation was solved on a computer and it was found that for most regions only two solutions branded β_1 and β_2 were admissible or significant. These two solutions are shown in Figure 5a for the unreversed case and in Figure 5b for the alternately reversed case. Let us compare these curves with the single mutual case. Notice first of all that the results for the single mutual case (see Figure 4a and 4b) seem to constitute the predominant part of the solutions for higher order mutuals. As for instance β_1 in Figure 6b is predominant in the lower part of the curve whereas β_2 might be considered as important above the resonance point kh = $\pi/2$. For the unreversed case, comparison between Figures 6a and 4a shows that single mutual solution essentially picks out the β_1 curve.

The problem of deciding which solution is more important than the other is not at all easy and in fact a satisfactory answer to this question may be found only by solving a source problem. However, the attenuation constant may be considered as a fair guide for choosing a predominant solution out of several different ones. This criterion may be misleading though because a particular mode, even though it has a relatively high attenuation may still be important close to the feed region and may strongly influence the far field pattern. All these considerations must be taken into account before deciding in the case of multimode propagation that a particular solution is most predominant and it is best to put in a word of explanation saying what one really means by that statement.

It is possible to conceive of many situations where it is difficult to pick out the predominant mode in a multimode operation. As a simple example let us reconsider Figure 6 a. It is certainly possible that both β_1 and β_2 are important in the resonant region and that the experimental measurements show a stronger influence of β_2 in this region. This is possibly why the measured curve show points in a region much slower than those predicted on the basis of β_1 solution alone. Experimental measurements reported by Hudock* on the monopole array exhibit the multimode type of propagation in that structure and his results also point to the fact that it is difficult to separate the multiple modes in the neighborhood of the resonance region.

We shall close this section with one further comment. Although we have not shown here the other two solutions which were obtained for the 3 mutual case, it seems that one of these extra solutions may play a significant part in the neightborhood of resonance. The third solution, may be excited with a comparable amplitude in this region and interfere with the β_1 solution (for the reversed case) to give a beat type of pattern which attenuates along the structure. This has been observed by Mayes and Ingerson[10] during their measurements on the dipole structure. At this stage, this explanation is merely a conjecture on our part, and further evidence is needed before the explanation can be considered as convincing.

However, one must not lose sight of one very important point in connection with this discussion. Even if we know that mode 1 and 3, say, are

* to be published

comparable in some region of frequency in the uniform periodic case, this does not enable us to say anything definite about the relative importance of mode 3 as compared to mode 1 or even the existence of mode 3, in the LP case. No satisfactory formulation is available as yet for answering these questions. This, in fact, is one of the limitations of the approach which uses the properties of the uniform periodic structure for predicting the performance of the corresponding LP one.

In concluding this section we observe that there is a fairly satisfactory correlation of the theoretical calculations with the experimental results. We note that multimode propagation exists in the periodically loaded transmission when the mutual terms are considered and that this has not been reported by the experimental observers except in a very small frequency range near the dipole resonance.

CONCLUSION

The Brillouin (k-β) diagrams for the dipole array have been calculated both for the unreversed and alternately reversed cases. The significance of the k-β diagram to the analysis of LP structures has been indicated. Theoretical calculations have been compared with experiments with good agreement. Multimode propagation is observed on the array structure; that this phenomenon may sometimes put a limitation on the simple perturbational approach of analyzing LP structure is pointed out.

ACKNOWLEDGEMENT

The authors would like to acknowledge the support of Hughes Aircraft Company through company sponsorship, and Wright Patterson Air Force Base, Ohio through Contract AF33(657)10474 for sponsoring the investigation reported in this paper. Numerical computation reported here were carried out at the Numerical Analysis Section of the Ground Systems Group, Hughes Aircraft Company. Thanks are particularly due to Dr. R. H. DuHamel, Head of the Electromagnetics Staff, Hughes Aircraft Company with whom one of the authors was associated during the summer of 1962. We are also indebted to E. Hudock, formerly of Antenna Laboratory, University of Illinois, for permission to use some results from his forthcoming Technical Report. Discussions with Professors G. A. Deschamps, P. E. Mayes and Messrs. S. Laxpati, P. E. Ingerson, E. Hudock, and other members of the Antenna Laboratory of the University of Illinois were always very helpful, and the authors are much indebted for what they learned from these discussions.

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FIGURE CAPTIONS

Figure 1a	Typical k-β diagrams for helix
Figure 1b	Typical k-β diagram for corrugated surface
Figure 1c	Typical k-β diagram for stub-loaded monopole array
Figure 2	Transmission line periodically loaded by a periodic loading circuit
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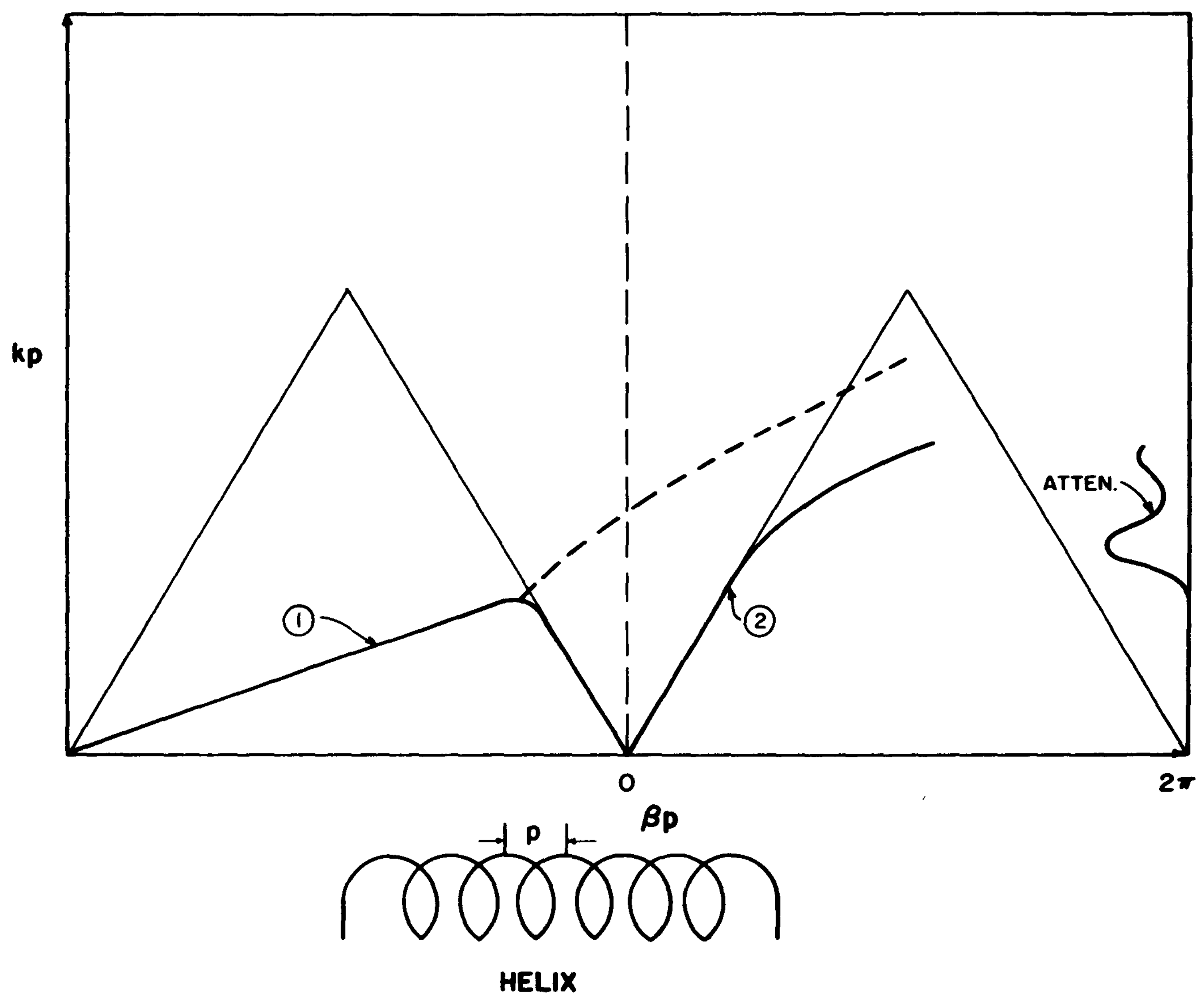
Figure 5	Transmission line voltage amplitude and phase curves for LP dipole array (theoretical) and comparison with experiment (Carrel)
Figure 6	Theoretical k-β diagrams for dipole arrays with three mutual impedances included - (a) unreversed case; (b) alternately reversed case


kp
ATTEN.
1
2
0
2π
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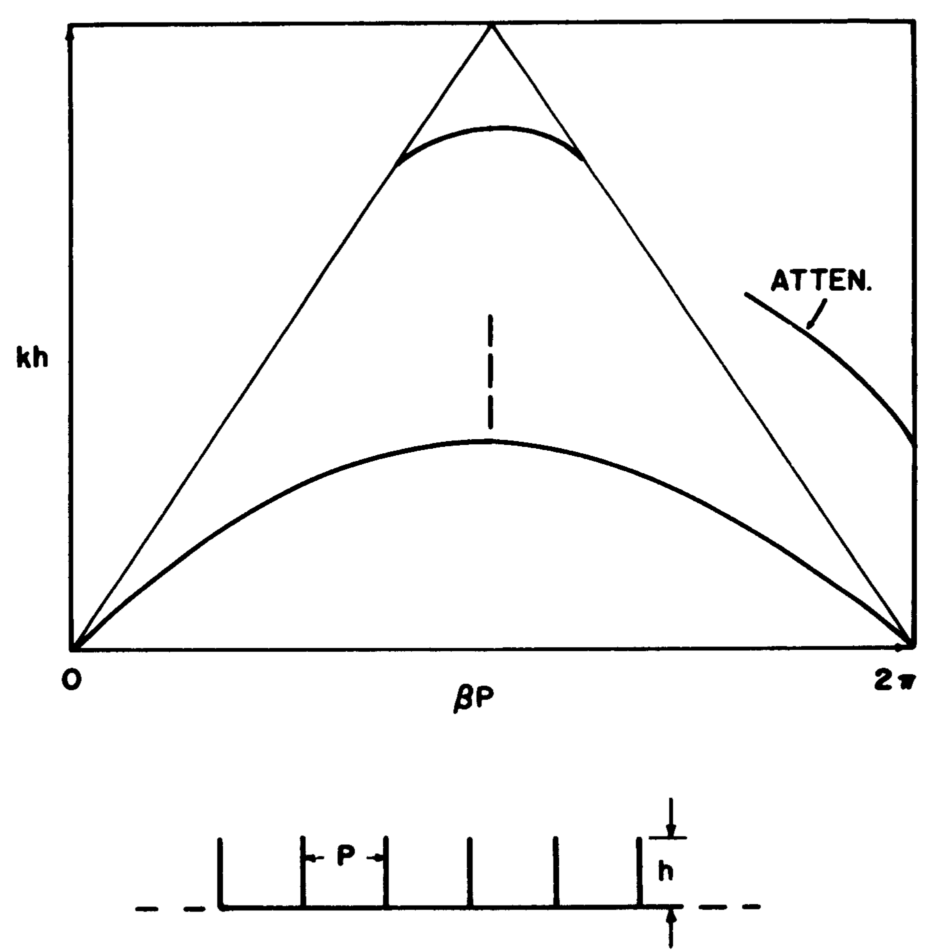
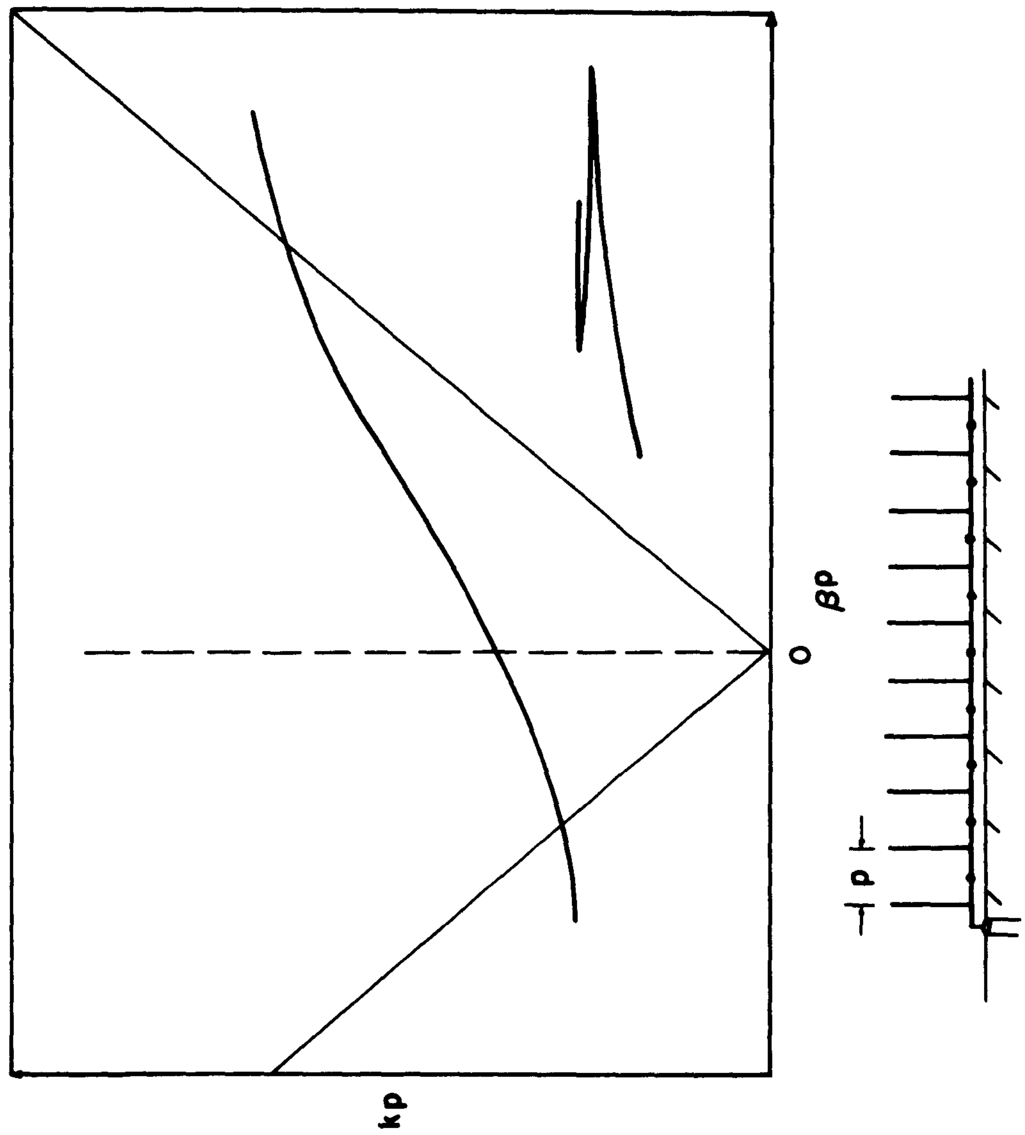
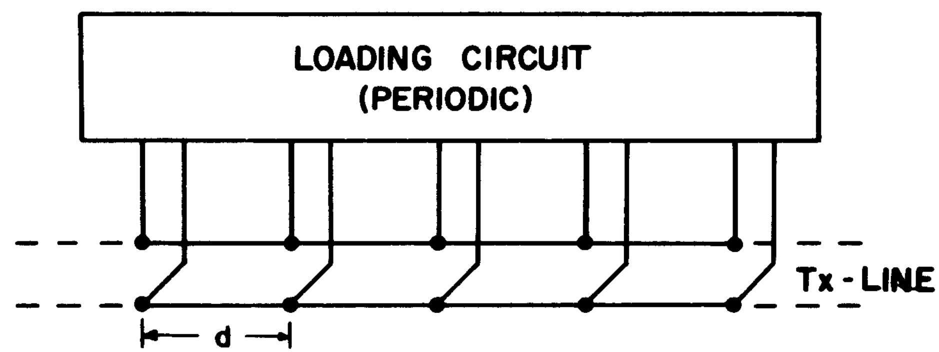
HELIX

CORRUGATED SURFACE



STUB LOADED MONOPOLES





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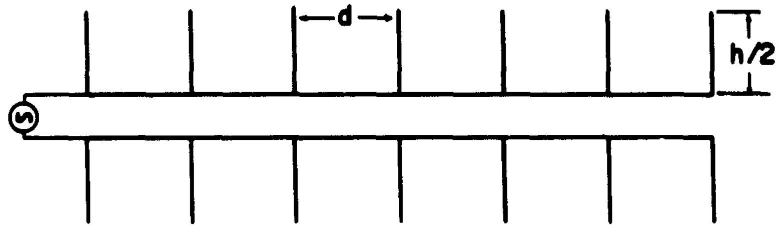
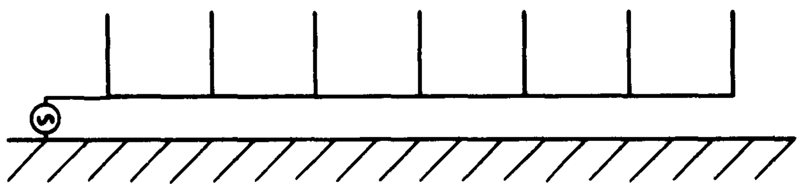
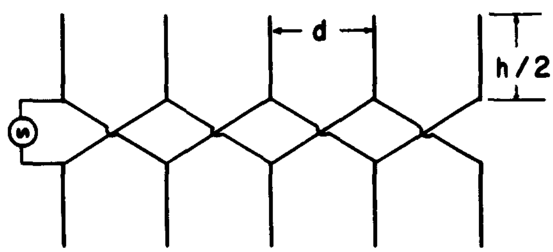


(a) DIPOLE ARRAY (UNREVERSED)

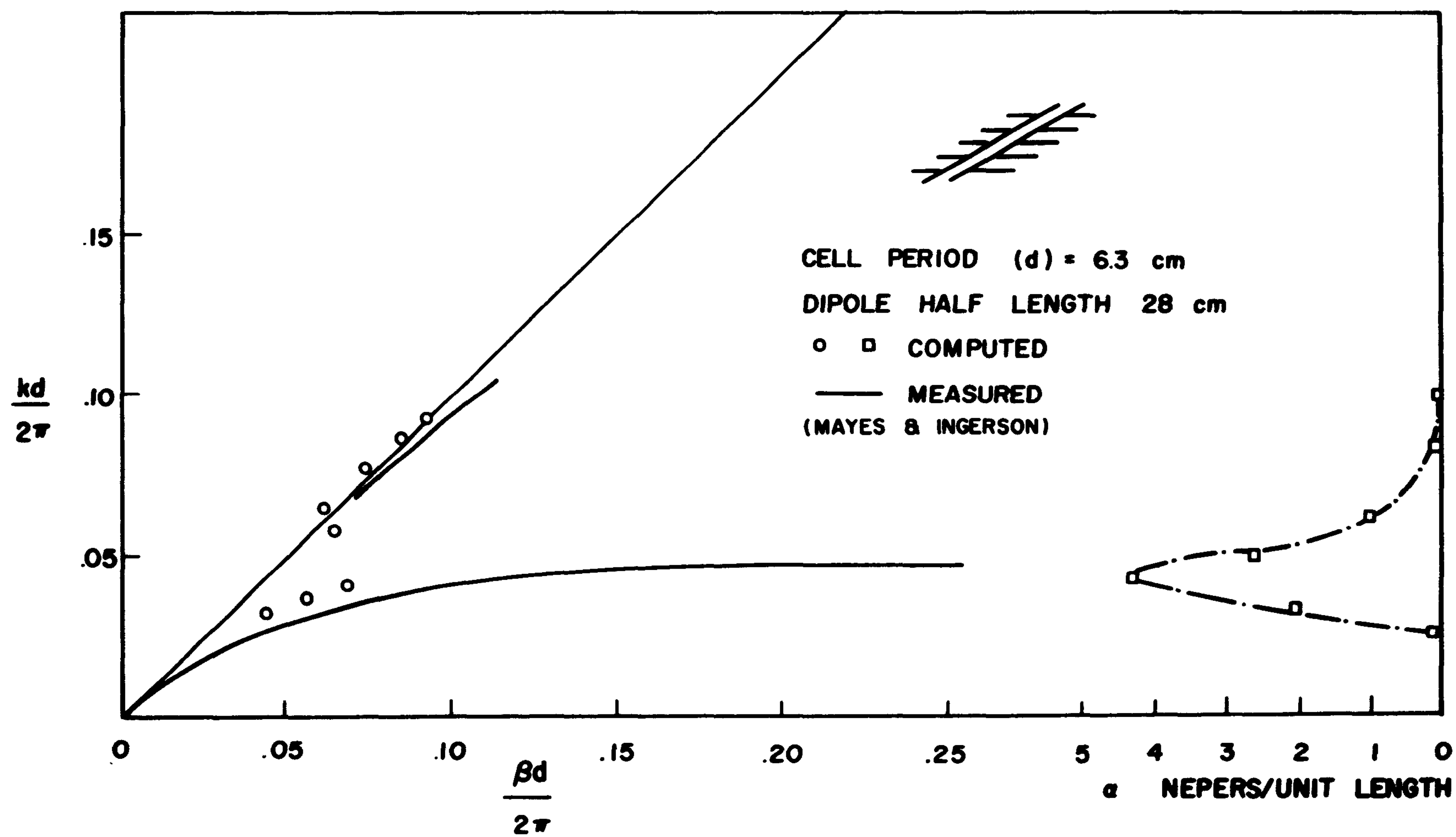


(b) MONOPOLE ARRAY

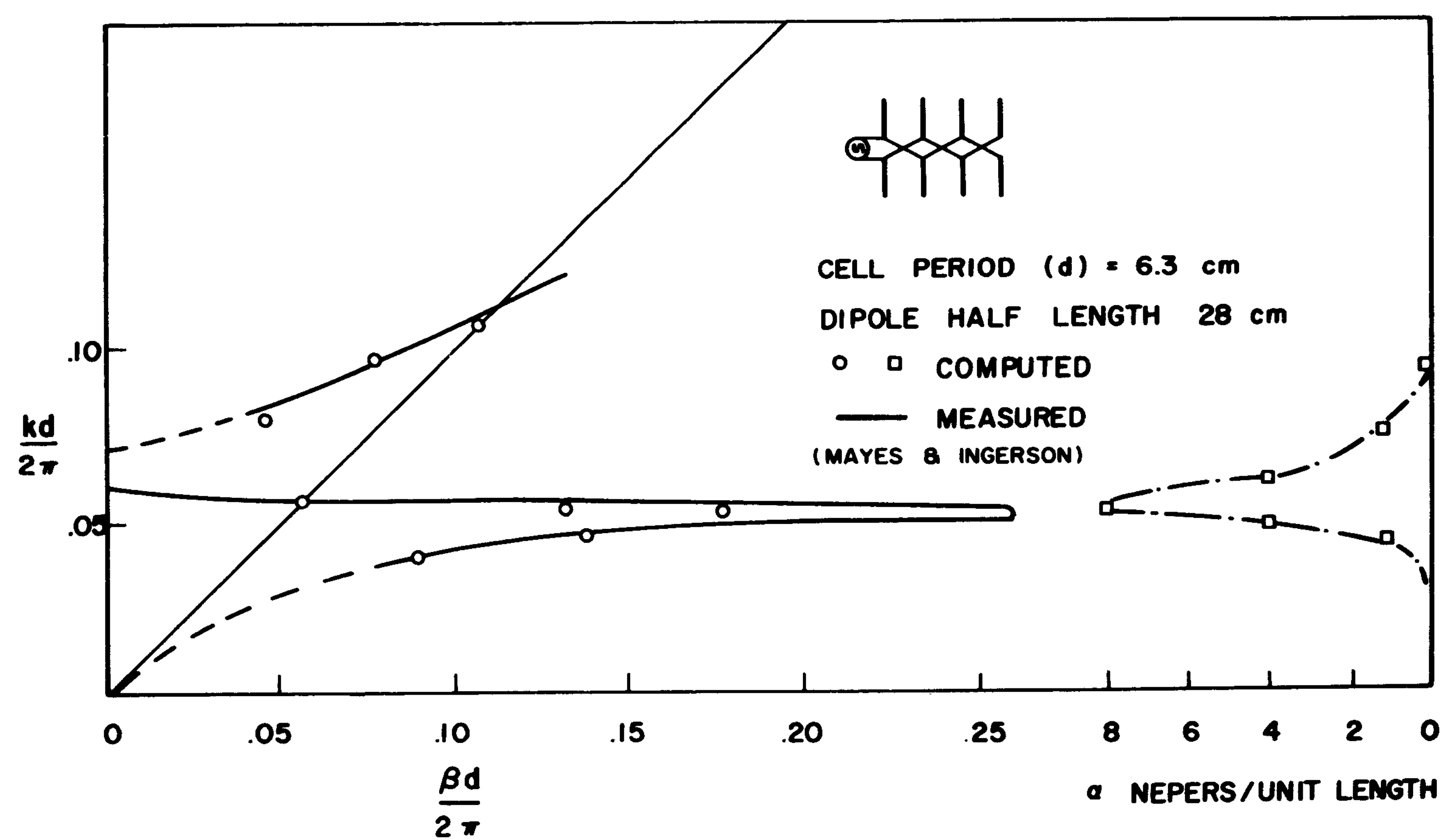


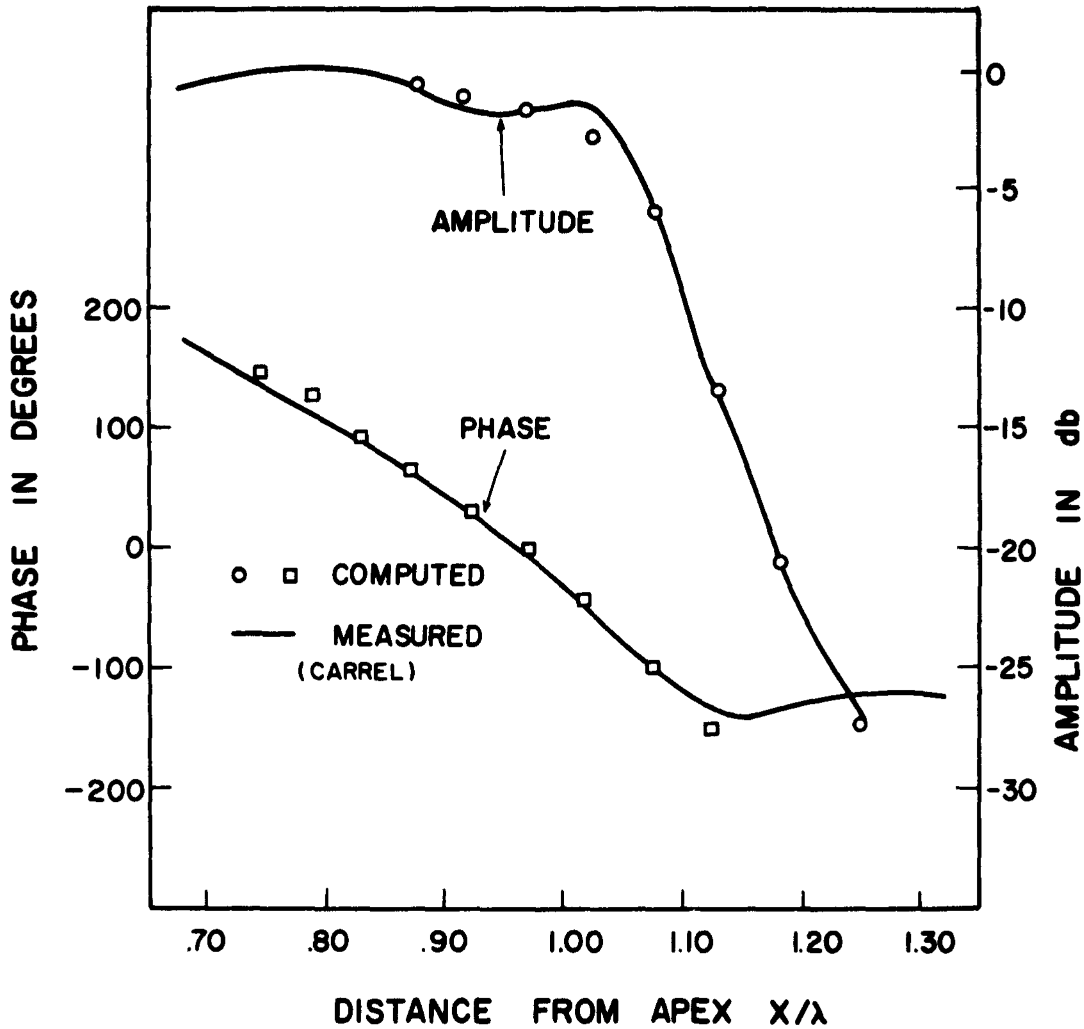
(c) DIPOLE ARRAY (ALTERNATELY REVERSED)

K - β DIAGRAM FOR DIPOLE ARRAY (UNREVERSED)

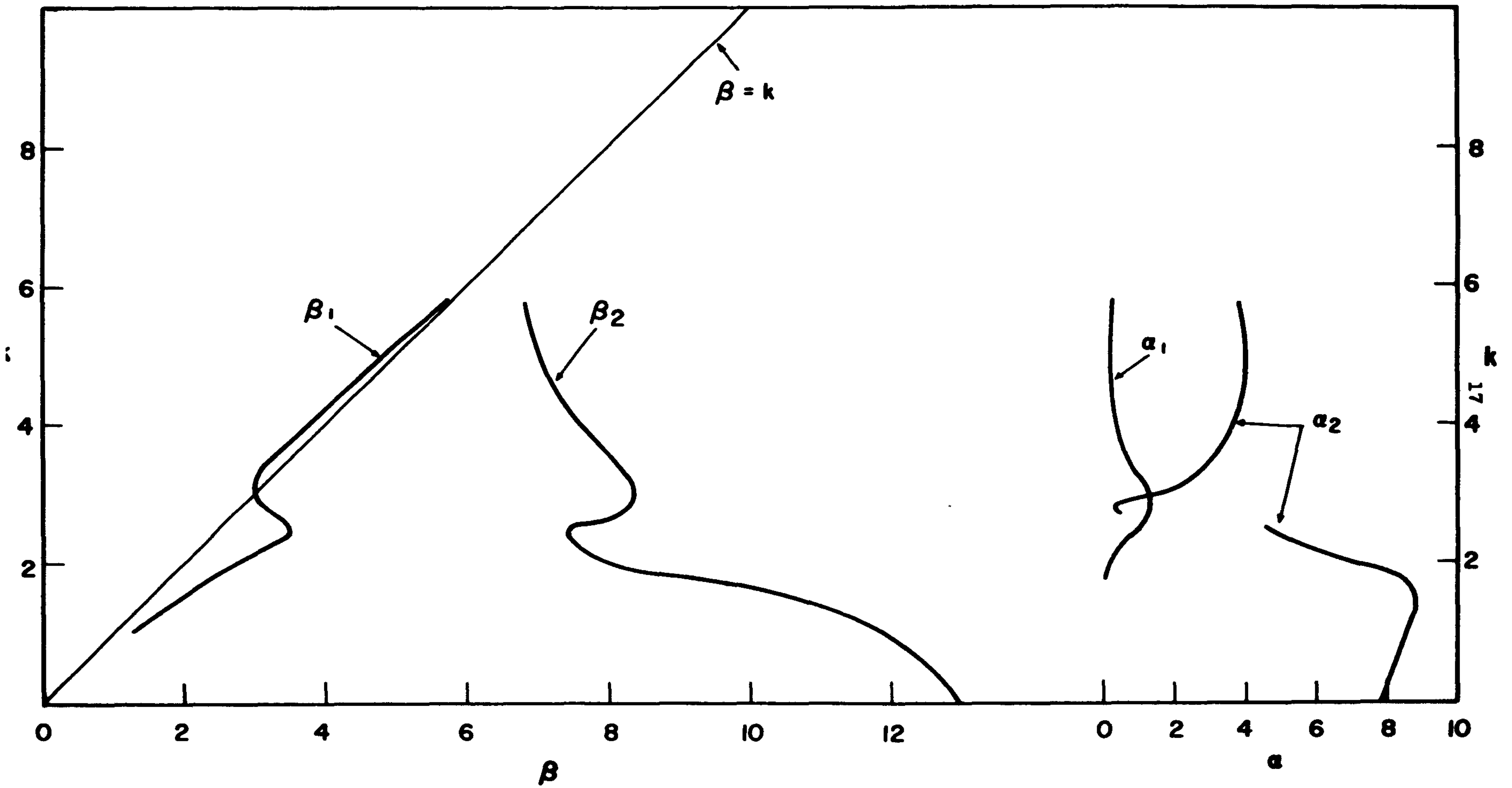


K-β DIAGRAM FOR DIPOLE ARRAY (ALTERNATELY REVERSED)

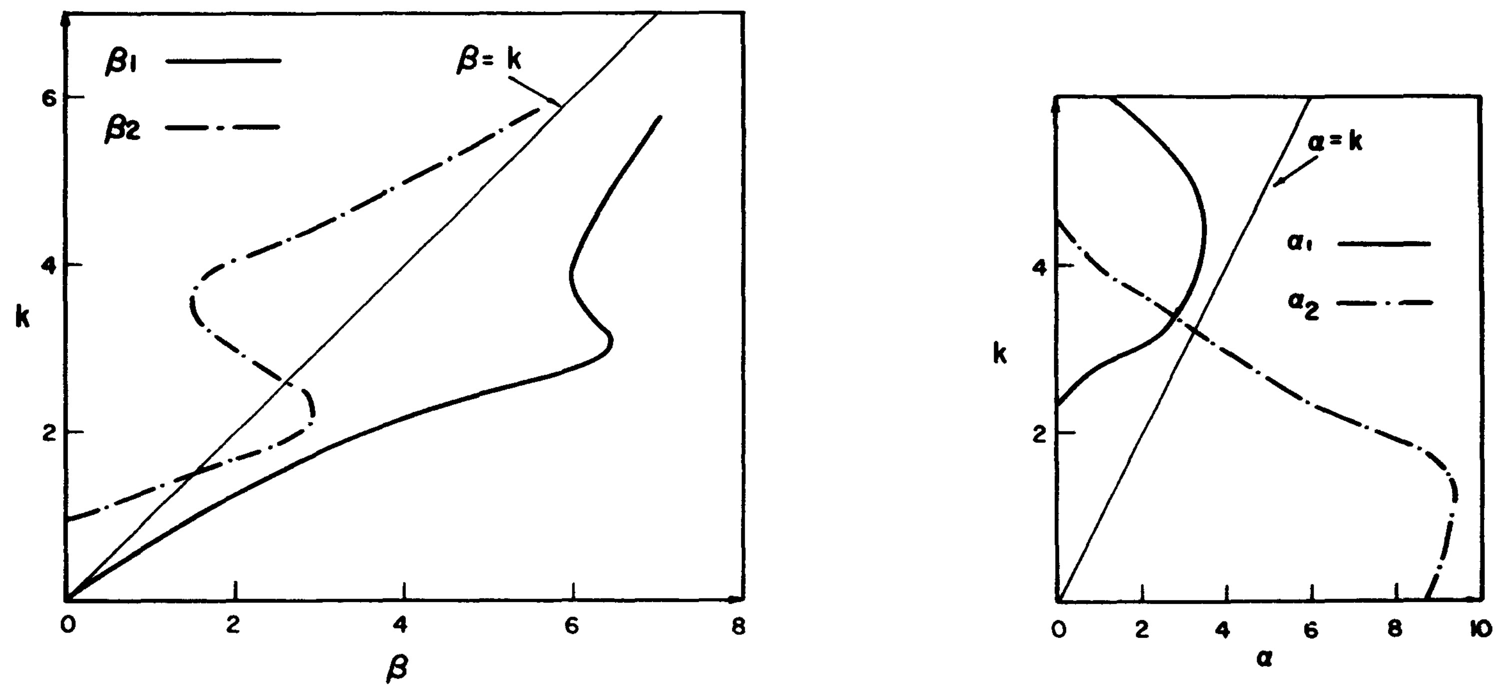




TRANSMISSION LINE VOLTAGE AMPLITUDE AND PHASE – LOG PERIODIC DIPOLE ARRAY



THREE MUTUAL CASE (UNREVERSED)



THREE MUTUAL CASE (REVERSED)

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Page	Column	Line	Reads	Should Read
1	2	8	Mittra	Mittra[4]
2	2	3 from bottom	Node 1	Node 0
2	2	last line	y_1-q	y_1 -q
3	1	2 after Equation (1)	Node 1	Node 0
4	2	4 after Equation (14)	where K (x) and Ci(x)	where Ci(x) and Si(x)
5	1	2 in last paragraph	Equation (15)	Equation (17)
5	1	2 from bottom	Equation (14)	Equation (16)
5	1	last line	Equation (15)	Equation (17)
5	2	1	Equation (14)	Equation (16)
6	1	8 from bottom	Figure 5a	Figure 6a
6	1	8 from bottom	Figure 5b	Figure 6b
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